



해군사관학교
REPUBLIC OF KOREA NAVAL ACADEMY

유 체 역 학

『21. 유동의 미분해석』

해군사관학교 공학처
기계시스템공학과

기계시스템공학교수 중위 노정환

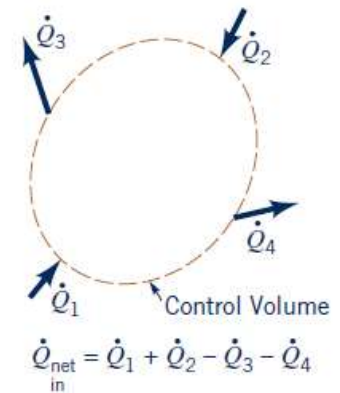
Review – Fluid Mechanics



❖ 1st Law of Thermodynamics

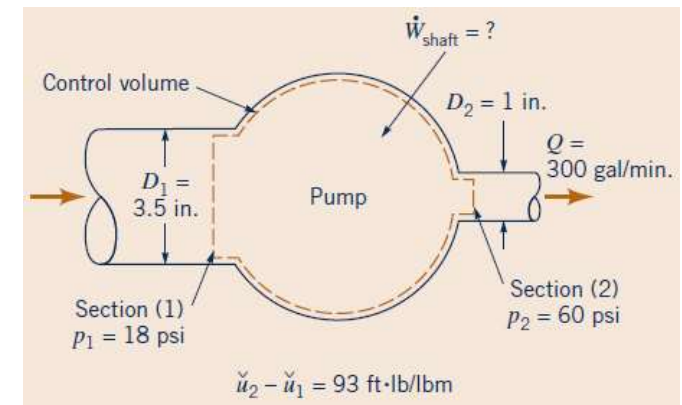
▪ The Energy Equation

$$\frac{\partial}{\partial t} \int_{cv} e \rho dV + \int_{cs} \left(\check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right) \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = \dot{Q}_{net \text{ in}} + \dot{W}_{shaft \text{ net in}}$$



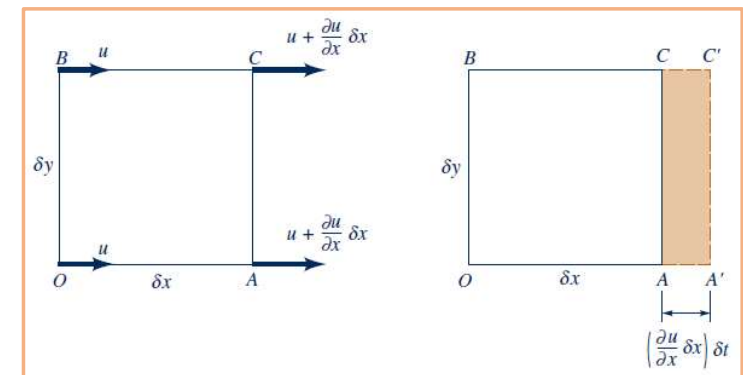
❖ Application of the Energy Equation

▪ One-dimensional energy equation



❖ Differential Analysis

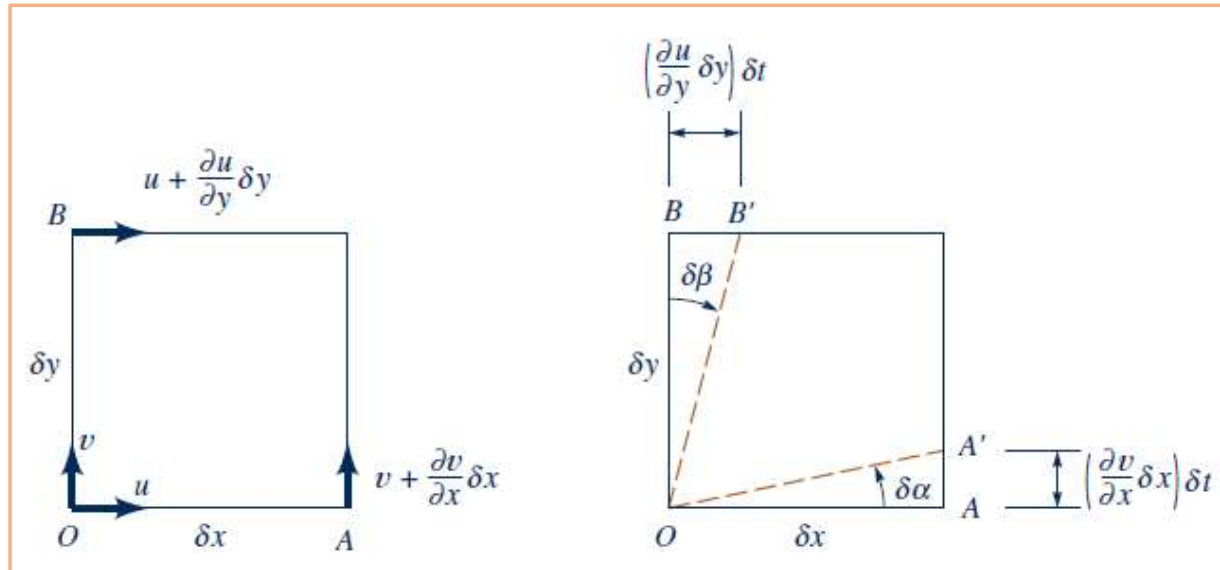
- Fluid Element Kinematics
- Linear Motion and Deformation



Differential Analysis



❖ Angular Motion and Deformation



❖ Vorticity

$$\zeta = 2\omega = \nabla \times \mathbf{V}$$

■ Irrotational Flow

$$\nabla \times \mathbf{V} = 0$$

$$\begin{aligned} \frac{1}{2} \nabla \times \mathbf{V} &= \frac{1}{2} \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} \\ &= \frac{1}{2} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) \hat{\mathbf{i}} + \frac{1}{2} \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) \hat{\mathbf{j}} + \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \hat{\mathbf{k}} \end{aligned}$$

Conservation of Mass



❖ Differential Form of Continuity Equation

$$\frac{\partial}{\partial t} \int_{cv} \rho dV + \int_{cs} \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = 0$$

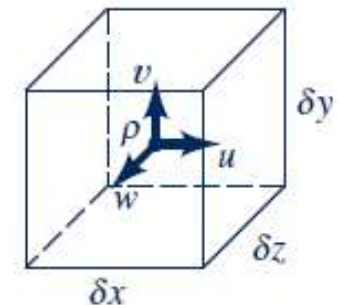
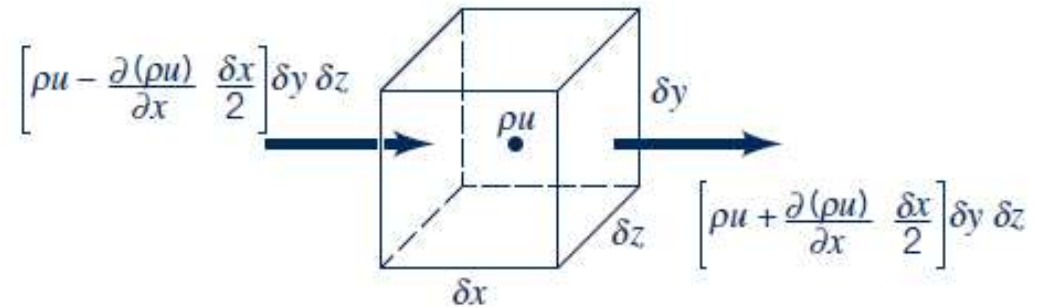
$$\frac{\partial}{\partial t} \int_{cv} \rho dV \approx \frac{\partial \rho}{\partial t} \delta x \delta y \delta z$$

$$\rho u|_{x+(\delta x/2)} = \rho u + \frac{\partial(\rho u)}{\partial x} \frac{\delta x}{2}$$

$$\rho u|_{x-(\delta x/2)} = \rho u - \frac{\partial(\rho u)}{\partial x} \frac{\delta x}{2}$$

Net rate of mass outflow in x direction = $\left[\rho u + \frac{\partial(\rho u)}{\partial x} \frac{\delta x}{2} \right] \delta y \delta z$

$$- \left[\rho u - \frac{\partial(\rho u)}{\partial x} \frac{\delta x}{2} \right] \delta y \delta z = \frac{\partial(\rho u)}{\partial x} \delta x \delta y \delta z$$



$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

Forces Acting on the Differential Element



❖ Conservation of Linear Momentum

$$\sum \mathbf{F}_{\text{contents of the control volume}} = \frac{\partial}{\partial t} \int_{\text{cv}} \mathbf{V} \rho dV + \int_{\text{cs}} \mathbf{V} \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA$$

$$\delta \mathbf{F} = \frac{D(\mathbf{V} \delta m)}{Dt} \quad \delta \mathbf{F} = \delta m \frac{D\mathbf{V}}{Dt} \quad \delta \mathbf{F} = \delta m \mathbf{a}$$

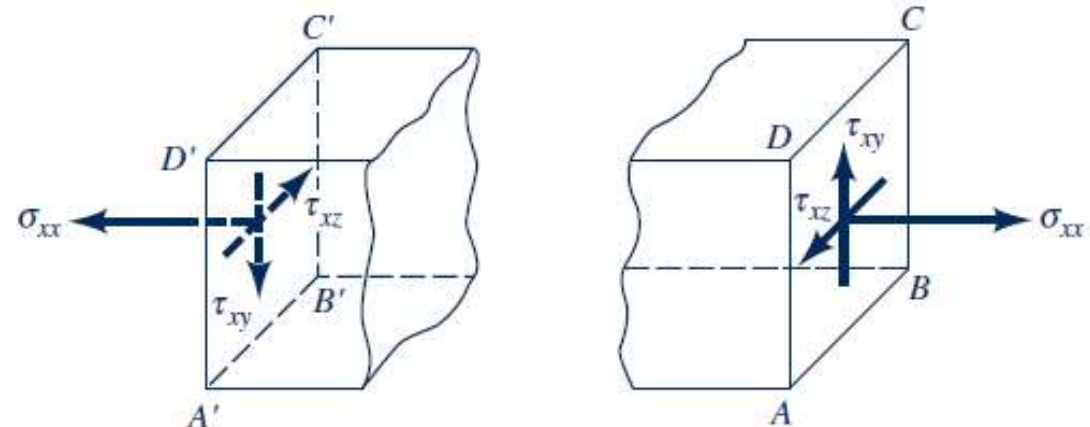
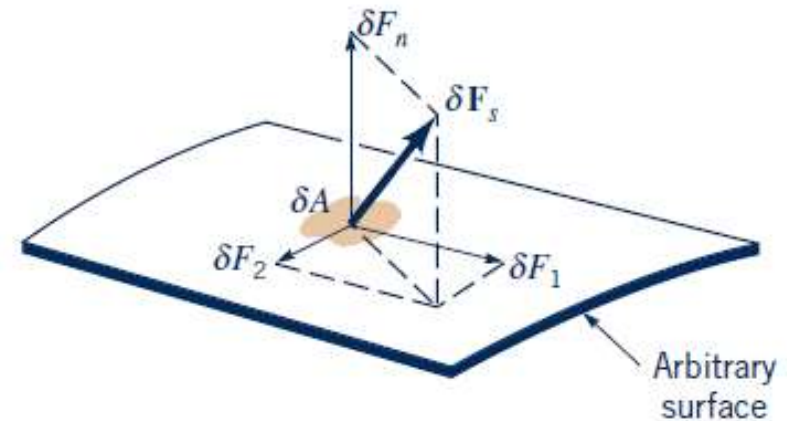
❖ Forces on the Element

$$\delta \mathbf{F} = \delta \mathbf{F}_s + \delta \mathbf{F}_b$$

$$\delta \mathbf{F}_b = \delta m \mathbf{g}$$

$$\delta F_{sx} = \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right) \delta x \delta y \delta z$$

$$\delta \mathbf{F}_s = \delta F_{sx} \hat{\mathbf{i}} + \delta F_{sy} \hat{\mathbf{j}} + \delta F_{sz} \hat{\mathbf{k}}$$



Equations of Motion



❖ Cauchy's Equation of Motion

$$\begin{aligned}\delta F_x = \delta m a_x & \quad \rho g_x + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = \rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) \\ \delta F_y = \delta m a_y & \quad \rho g_y + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} = \rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) \\ \delta F_z = \delta m a_z & \quad \rho g_z + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} = \rho \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right)\end{aligned}$$

❖ Inviscid Flow – Euler's Equation

$$\begin{aligned}\rho g_x - \frac{\partial p}{\partial x} &= \rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) \\ \rho g_y - \frac{\partial p}{\partial y} &= \rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) \\ \rho g_z - \frac{\partial p}{\partial z} &= \rho \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right)\end{aligned}$$

Viscous Fluid Flow

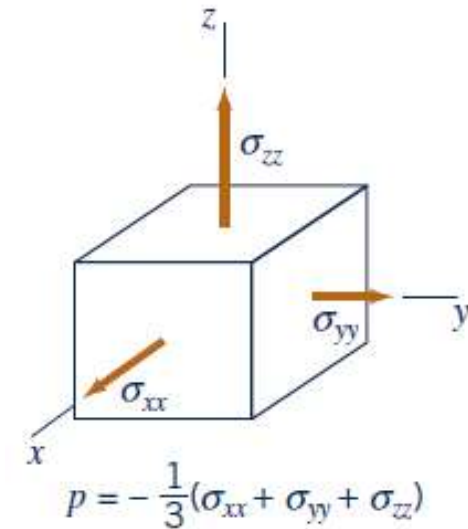


❖ Constitutive Equation

$$\sigma_{xx} = -p + 2\mu \frac{\partial u}{\partial x} \quad \tau_{xy} = \tau_{yx} = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$$

$$\sigma_{yy} = -p + 2\mu \frac{\partial v}{\partial y} \quad \tau_{yz} = \tau_{zy} = \mu \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)$$

$$\sigma_{zz} = -p + 2\mu \frac{\partial w}{\partial z} \quad \tau_{zx} = \tau_{xz} = \mu \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right)$$



❖ The Navier – Stokes Equations

(x direction)

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial x} + \rho g_x + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

(y direction)

$$\rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = -\frac{\partial p}{\partial y} + \rho g_y + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right)$$

(z direction)

$$\rho \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = -\frac{\partial p}{\partial z} + \rho g_z + \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right)$$

Q n A

대양해군

★통-111호 / 010-9388-9369